MMC controllers for ultra low harmonic distortion

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Basic approach

• Comprehensive study on the behavior of MMCs in Frequency domain.

• **Iterative algorithm** to obtain control signals as fast as possible.

• Controller **based on FFT**, that’s readily available in digital domain.
The proposed system described above minimizes the harmonic distortion of the output current by adjusting the higher harmonics in control signals. It adopts an iterative algorithm in finding the optimum control signals. N<16 has been found to be enough for most of cases.
--- Test circuits ---
Test circuit I (LCR series circuit)

- The simplest conceivable circuit for the control
- continuous-time approximation to check THD
- This simple study is applicable to controls of other complicated MMC circuits.

\[ E = 100 \text{ [V]} \]
\[ L = 0.5/1.0/2.0 \text{ [mH]} \]
\[ C = 0.5/1.0/2.0 \text{ [mF]} \]
\[ R = 10 \text{ [Ω]} \]
\[ f = 50 \text{ [Hz]} \]
Test circuit II (The simplest MMC)

Performed a transient simulation (LTSpice IV) to check THD.

\[ E = 100 \quad [V] \]
\[ L_a = 0.5 / 1.0 / 2.0 \quad [mH] \]
\[ \Delta L_a = 0 \quad [mH] \]
\[ L_l = 10 \quad [mH] \]
\[ C = 0.25 / 0.5 / 1.0 \quad [mF] \]
\[ R = 10 \quad [\Omega] \]
\[ r = 0.3 \quad [\Omega] \]
\[ f = 50 \quad [Hz] \]
--- Construction of the models ---
The derivatives of the cap voltage and the output current is expressed in a linear formation.

\[
\frac{d}{dt} \begin{pmatrix} v_c(t) \\ i_{out}(t) \end{pmatrix} = \begin{pmatrix} \alpha(t)i_{out}(t) \\ \frac{1}{C}(E - \alpha(t)v_c(t) - Ri_{out}(t)) \end{pmatrix} = \begin{pmatrix} 0 & \alpha(t) \\ -\frac{\alpha(t)}{C} & -\frac{R}{L} \end{pmatrix} \begin{pmatrix} v_c(t) \\ i_{out}(t) \end{pmatrix} + \frac{1}{L} \begin{pmatrix} 0 \\ E \end{pmatrix}
\]
Continuous-time model of test circuit I (2)

\[
\frac{d}{dt} \left( \begin{array}{c} v_c(t) \\ i_{out}(t) \end{array} \right) = \left( \begin{array}{c} \alpha(t) i_{out}(t) \\ \frac{1}{C} \left( E - \alpha(t)v_c(t) - Ri_{out}(t) \right) \end{array} \right) = \left( \begin{array}{cc} 0 & \frac{\alpha(t)}{C} \\ -\frac{\alpha(t)}{L} & -\frac{R}{L} \end{array} \right) \left( \begin{array}{c} v_c(t) \\ i_{out}(t) \end{array} \right) + \frac{1}{L} \left( \begin{array}{c} 0 \\ E \end{array} \right)
\]

Periodic case

\[ i_{out} \]
\[ v_c \]

Phase space is helpful for our intuitive understandings of MMC dynamics.

Transient case

\[ i_{out} \]
\[ v_c \]

Cap voltage control

Start up
Continuous-time model of test circuit II (1)

However complicated the circuit would be, the dynamic system can be expressed in linear formation (matrices).
Continuous-time model of test circuit II (2)

\[
\frac{d}{dt} \begin{pmatrix} \vec{v}_c(t) \\ \vec{i}(t) \end{pmatrix} = \begin{pmatrix} \frac{\alpha(t)\vec{i}(t)}{C} \\ L^{-1}(E - \alpha(t)\vec{v}_c(t) - R\vec{i}(t)) \end{pmatrix} = \begin{pmatrix} 0 & \frac{\alpha(t)}{C} \\ -L^{-1}\alpha(t) & -L^{-1}R \end{pmatrix} \begin{pmatrix} \vec{v}_c(t) \\ \vec{i}(t) \end{pmatrix} + L^{-1}\begin{pmatrix} \vec{0} \\ \vec{E} \end{pmatrix}
\]

\[
\vec{v}_c(t) = \begin{pmatrix} v_{ch}(t) \\ v_{cl}(t) \end{pmatrix}, \quad \vec{i}(t) = \begin{pmatrix} i_h(t) \\ i_l(t) \end{pmatrix}, \quad \vec{E} = \begin{pmatrix} E \\ E \end{pmatrix}
\]

\[
R = \begin{pmatrix} r + R & -R \\ -R & r + R \end{pmatrix}, \quad L = \begin{pmatrix} L_a + L_l & -L_l \\ -L_l & L_a + L_l \end{pmatrix}, \quad \alpha(t) = \begin{pmatrix} \alpha_h(t) & 0 \\ 0 & \alpha_l(t) \end{pmatrix}
\]

It’s very similar to the continuous-time model of test circuit I (simple LCR series circuit) !!
--- Details of the iterative algorithm ---
Iterative algorithm for test circuit I (1)

\[ \frac{d}{dt} \left( \begin{array}{c} v_c(t) \\ i_{out}(t) \end{array} \right) = \left( \begin{array}{c} \frac{\alpha(t)i_{out}(t)}{C} \\ \frac{1}{L} \left( E - \alpha(t)v_c(t) - R_i_{out}(t) \right) \end{array} \right) = \left( \begin{array}{cc} 0 & \frac{\alpha(t)}{C} \\ -\frac{\alpha(t)}{L} & -\frac{R}{L} \end{array} \right) \left( \begin{array}{c} v_c(t) \\ i_{out}(t) \end{array} \right) + \frac{1}{L} \left( \begin{array}{c} 0 \\ E \end{array} \right) \]

\[ v_c(t) = \sum_{k=-\infty}^{\infty} \tilde{v}_c^{(k)} e^{jk\omega t} \quad i_{out}(t) = \sum_{k=-\infty}^{\infty} \tilde{i}_{out}^{(k)} e^{jk\omega t} \quad \alpha(t) = \sum_{k=-\infty}^{\infty} \tilde{\alpha}^{(k)} e^{jk\omega t} \]

Calculate the Fourier transform of the fundamental equation to find the optimum \( \alpha(t) \) for non-distorted \( i_{out}(t) \).
Iterative algorithm for test circuit I (2)

<Fundamental equations for non-distorted $i_{\text{out}}(t)$>

$$
\tilde{v}_c^{(0)}\tilde{\alpha}^{(k)} + f\left(\{\tilde{\alpha}^{(n)}\}, \infty, k\right) = \begin{cases} 
E - R\tilde{i}_{\text{out}}^{(0)} & (k = 0) \\
-(j\omega L + R)\tilde{i}_{\text{out}}^{(1)} & (k = 1) \\
-(j\omega L + R)\tilde{i}_{\text{out}}^{(-1)} & (k = -1) \\
0 & (|k| \geq 2)
\end{cases} \quad \text{Condition 1}
$$

$$
\tilde{\alpha}^{(0)} \frac{E}{R} = \tilde{v}_c^{(0)} \sum_{k=-\infty}^{\infty} \frac{|\tilde{\alpha}^{(k)}|^2}{jk\omega L + R} + \sum_{k=-\infty}^{\infty} \frac{\tilde{\alpha}^{(-k)}}{jk\omega L + R} f\left(\{\tilde{\alpha}^{(n)}\}, \infty, k\right) \quad \text{Condition 2}
$$

where

$$
f\left(\{\tilde{\alpha}^{(n)}\}, N, p\right) = \sum_{m=-N/2}^{-1} \tilde{\alpha}^{(p-m)} \frac{\tilde{i}_{\text{out}}^{(0)}\tilde{\alpha}^{(m)} + \tilde{i}_{\text{out}}^{(-1)}\tilde{\alpha}^{(m+1)} + \tilde{i}_{\text{out}}^{(1)}\tilde{\alpha}^{(m-1)}}{jm\omega C} + \sum_{m=1}^{N/2} \tilde{\alpha}^{(p-m)} \frac{\tilde{i}_{\text{out}}^{(0)}\tilde{\alpha}^{(m)} + \tilde{i}_{\text{out}}^{(-1)}\tilde{\alpha}^{(m+1)} + \tilde{i}_{\text{out}}^{(1)}\tilde{\alpha}^{(m-1)}}{jm\omega C}
$$
Iterative algorithm for test circuit I (3)

Iterative algorithm for test circuit I

\[
(\tilde{v}_c^{(0)}, \tilde{i}_{out}^{(1)}) \Rightarrow (\tilde{\alpha}^{(0)}, \tilde{\alpha}^{(1)}, \tilde{i}_{out}^{(0)})
\]

Input parameters

Initial values

\[
\tilde{v}_c^{(0)} \tilde{\alpha}_0^{(k)} = \begin{cases} 
E - R\tilde{i}_{out}^{(0)} & (k = 0) \\
-(j\omega L + R)\tilde{i}_{out}^{(1)} & (k = 1) \\
-(j\omega L + R)\tilde{i}_{out}^{(1)} & (k = -1) \\
0 & (|k| \geq 2)
\end{cases}
\]

Equation for the iteration

\[
\tilde{v}_c^{(0)} \tilde{\alpha}_{q+1}^{(k)} + f(\{\tilde{\alpha}_q^{(n)}\}, N, k) = \begin{cases} 
E - R\tilde{i}_{out}^{(0)} & (k = 0) \\
-(j\omega L + R)\tilde{i}_{out}^{(1)} & (k = 1) \\
-(j\omega L + R)\tilde{i}_{out}^{(1)} & (k = -1) \\
0 & (|k| \geq 2)
\end{cases}
\]

\((q+1)\)'th complex series \(\{\tilde{\alpha}_{q+1}^{(n)}\}\) can be obtained by calculating \(f(\{\tilde{\alpha}_q^{(n)}\}, N, k)\)
Iterative algorithm for test circuit II (1)

**Fundamental equation of test circuit II**

\[
\frac{d}{dt} \left( \tilde{v}_c(t) \right) = \left( \begin{array}{c} \frac{\alpha(t) \tilde{i}(t)}{C} \\ L^{-1} \left( E - \alpha(t) \tilde{v}_c(t) - R \tilde{i}(t) \right) \end{array} \right) = \left( \begin{array}{cc} 0 & \alpha(t) \\ -L^{-1} \alpha(t) & -L^{-1}R \end{array} \right) \left( \begin{array}{c} \tilde{v}_c(t) \\ \tilde{i}(t) \end{array} \right) + L^{-1} \left( \begin{array}{c} \tilde{0} \\ \tilde{E} \end{array} \right)
\]

**Fourier transformation**

\[
\tilde{v}_c(t) = \sum_{k=-\infty}^{\infty} \tilde{v}_c^{(k)} e^{jk\omega t} \quad \tilde{i}(t) = \sum_{k=-\infty}^{\infty} \tilde{i}^{(k)} e^{jk\omega t} \quad \alpha(t) = \sum_{k=-\infty}^{\infty} \alpha^{(k)} e^{jk\omega t}
\]

Fourier transform to find the optimum \( \alpha(t) \) for non-distorted \( \tilde{i}_{out}(t) \).
Iterative algorithm for test circuit II (2)

\[
\begin{align*}
\left( \tilde{v}_c(0), \tilde{i}_{out}(1) \right) & \Rightarrow \left( \tilde{\alpha}(0), \tilde{\alpha}(1), \tilde{i}_{out}(0) \right) \\
\tilde{v}_c(0)\tilde{\alpha}^{(k)} + f\left( \{\tilde{\alpha}^{(n)}\}, N, k \right) &= \\
&= \begin{cases} 
\tilde{E} - \mathbf{R}\tilde{i}^{(0)} & (k = 0) \\
-(j\omega\mathbf{L} + \mathbf{R})\tilde{i}^{(1)} & (k = 1) \\
-(-j\omega\mathbf{L} + \mathbf{R})\tilde{i}^{(-1)} & (k = -1) \\
0 & (|k| \geq 2)
\end{cases}
\end{align*}
\]

where

\[
f\left( \{\tilde{\alpha}^{(n)}\}, N, p \right) = \sum_{m=-N/2}^{-1} \tilde{\alpha}^{(p-m)} \tilde{i}^{(0)}\tilde{\alpha}^{(m)} + \tilde{i}^{(-1)}\tilde{\alpha}^{(m+1)} + \tilde{i}^{(1)}\tilde{\alpha}^{(m-1)} + \sum_{m=1}^{N/2} \tilde{\alpha}^{(p-m)} \tilde{i}^{(0)}\tilde{\alpha}^{(m)} + \tilde{i}^{(-1)}\tilde{\alpha}^{(m+1)} + \tilde{i}^{(1)}\tilde{\alpha}^{(m-1)}
\]
--- Simulation results ---
Simulation results of test circuit I (1)

<THD vs # of harmonics>

\[ L = 0.5/1.0/2.0 \quad [\text{mH}] \]
\[ C = 0.5/1.0/2.0 \quad [\text{mF}] \]
\[ \tilde{v}_c^{(0)} = 130 \quad [\text{V}] \]
\[ I^{(1)} = 2.5 \quad [\text{A}] \]

We have only to optimize 8-16 harmonics in control signals to achieve sufficient THD performance.
Simulation results of test circuit I (2)

\[ \text{THD vs iteration count} \]

\[ \begin{align*}
L &= 0.5/1.0/2.0 \quad [\text{mH}] \\
C &= 0.5/1.0/2.0 \quad [\text{mF}] \\
\tilde{v}_c^{(0)} &= 130 \quad [\text{V}] \\
I^{(1)} &= 2.5 \quad [\text{A}] 
\end{align*} \]

• Required iteration count is less than 8 in typical design.
• Less capacitance tends to need more iteration counts.
Simulation results of test circuit I (3)

<dependence on $I^{(1)}$>

$L = 2.0$ [mH]

$C = 0.5$ [mF]

$\hat{v}_c^{(0)} = 110$ [V]

$I^{(1)} = 1.0/1.5/2.0/$

$2.5/3.0$ [A]

- Trajectories for 5 different output currents.
- Applicable to power controls without any degradation of THD.
Simulation results of test circuit I (4)

<dependence on $\tilde{v}_c^{(0)}$>

$L = 2.0 \quad [\text{mH}]$

$C = 0.5 \quad [\text{mF}]$

$I^{(1)} = 2.5 \quad [\text{A}]$

$\tilde{v}_c^{(0)} = 110 / 120 / 130$

$\quad / 140 / 150 \quad [\text{V}]$

- Trajectories for 5 different cap voltages.
- Applicable to cap voltage recovery (restart/mode-switch) without any degradation of THD.
Simulation results of test circuit II (1)

\[ L_a = 0.5 / 1.0 / 2.0 \quad [\text{mH}] \]
\[ C = 0.5 / 1.0 / 2.0 \quad [\text{mF}] \]
\[ \tilde{\nu}_{ch}^{(0)} = \tilde{\nu}_{cl}^{(0)} = 150 \quad [\text{V}] \]
\[ I^{(1)} = 4 \quad [\text{A}] \]

- Required iteration count is less than 8 in typical design.
- Test circuit II exhibits better THD results compared to test circuit I with the same cap values.
Simulation results of test circuit II (2)

<Spice simulation (1) – test bench – >

- Half bridge MMC inverter
- One cap for each arm for simplicity
- Switching freq: 12.8[kHz]
- PWM resolution: 100 [ns]
- Control signals @ 7’th iteration

\[ L_a = 2.0 \text{ [mH]} \]
\[ C = 0.25 \text{ [mF]} \]
\[ \tilde{v}_{ch}^{(0)} = \tilde{v}_{cl}^{(0)} = 200 \text{ [V]} \]
\[ I^{(1)} = 4 \text{ [A]} \]

Probe the current

Inductive load
Power factor ~ 0.95
Simulation results of test circuit II (3)

<Spice simulation (2) – output current – >

[transient waveform of the output current]

[FFT result of the output current]

- No spurious @50*n [Hz]
- Carrier leakage @ 12.8*n [kHz]
- THD is less than -80 [dB]
Simulation results of test circuit II (3)

<Spice simulation (3) – Before/After the correction→

Iteration count: 0
THD: -30.5 [dB]

Iteration count: 3
THD: -79.0 [dB]
Summary

• THD has been drastically reduced by adjusting a few harmonics in control signals.

• An iterative method has been found to be an effective approach to get numerical solutions of our complicated equations as fast as possible.

• This approach is applicable to any optimization in periodic controls.
--- Future prospects ---
Future prospects (1)

<Periodic control>

- Optimization of control signals including any non-ideal effects.
- For example, skew/device mismatch/dead time/clock feed through/non-linear load, etc.
Future prospects (2)

<Transient control>

- Optimization of evaluation functions in transient controls.
- For example, quick start-up/mode-switch/silent recovery from accidents.

Optimum transient control depending on the situations.
Future prospects (3)

• Optimization of other characteristics of MMCs in periodic/transient operations.

• Proposals of Digital controllers estimating and correcting any conceivable non-ideal factors in analog portions.

• Comprehensive studies on controls of various inverters/converters in periodic/transient operations.
--- Appendix ---
Simulation results of test circuit II

<Spice simulation of test circuit II – FFT results (1) – >

Iteration count: 0
THD: -30.5 [dB]

Iteration count: 1
THD: -50.2 [dB]
Simulation results of test circuit II

Spice simulation of test circuit II – FFT results (2) →

Iteration count: 2
THD: -70.8 [dB]

Iteration count: 3
THD: -79.0 [dB]
Simulation results of test circuit II

<Spice simulation of test circuit II – Cap voltages – >

[Cap voltages]

[Current in upper arm]

~24 [V]

Circulation Current
~0.5 [A]